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Impact Point Dispersion due to Spin Reversal

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Introduction

IT has been recognized that small re-entry vehicles can experience uncontrolled anomalous roll motions during reentry. The principal deleterious effect of these motions is the occurrence of a roll resonance or spin-through-zero roll-rate condition with attendant unacceptably large lateral loads and/or large impact point dispersions. To eliminate the possibility of encountering roll resonance, control of the roll rate to subresonant rates has been suggested. However, some candidate control schemes do not avoid the possibility of experiencing spin rate reversal. The purpose of this Note is to assess analytically the magnitudes of the resulting dispersions and their sensitivity to the variation of trajectory and body parameters. Also, the differences between the present analysis and the more restrictive previous work by Fuess[1] and Crenshaw and Tessitore[2] are indicated.

Analysis

For small roll rates, and away from resonance, the trim angle of attack in a body fixed coordinate system can be approximated by the complex number

$$\delta = \beta_t + i\alpha_t \tag{1}$$

where α_t and β_t are the components of the static trim angle of attack. Moreover, the variation of the roll rate, p, with time is given by

$$dp/dt = (qAd/I_x)C_l \tag{2}$$

where q is the dynamic pressure, A is the body cross-sectional area, d is the body diameter, I_x is the axial moment of inertia, and C_l is the roll moment coefficient and is given by[3]

$$C_l = \frac{c\alpha_t}{d} - \frac{c}{d}\,\Delta C_{N0}\cos\lambda + C_{lf}\,\frac{pd}{2V} + C_{l\theta}\,\theta + C_{lp}\,\frac{pd}{2V} \tag{3}$$

The first term on the right-hand side of Eq. (3) is due to the combination of a center of gravity (c.g.) offset and a trim δ. Without loss of generality, the c.g. offset plane is taken orthogonal to the plane of the α_t component of the trim δ. The second term is caused by a normal force component, ΔC_{N0}, which together with its orientation, λ, are independent of δ. This normal force is caused by the geometric nose asymmetrics. The third term represents the roll damping due to friction, while the fifth term represents the roll damping due to grooves in the body surface. The fourth term represents the ablation-grooving roll driving moment.

If the dispersion due to roll reversal is denoted by the complex number $z = z_\beta + iz_\alpha$, then

$$m\,\frac{d^2z}{dt^2} = Le^{iP}, \qquad P = \int_0^t p(\tau)d\tau \tag{4}$$

where m is the body mass, P is the roll orientation, and L is the lift force and approximately given by

$$L = qA\delta(C_{N\alpha} - C_A) \tag{5}$$

where $C_{N\alpha}$ is the normal force derivative per radian and C_A is the axial force coefficient. Integrating once Eq. (4), and separating real and imaginary parts lead to

$$\dot{z}_\beta = \int_0^t \frac{L}{m}\,(\beta_t\cos P - \alpha_t\sin P)dt \tag{6}$$

$$\dot{z}_\alpha = \int_0^t \frac{L}{m}\,(\beta_t\sin P + \alpha_t\cos P)dt \tag{7}$$

For large roll rates, the integrals in Eqs. (6) and (7) average out, and hence most of the contribution comes from the neighborhood of small roll rates, especially near zero roll rate if it exists. Thus, L and C_l can be taken to be constants, and equal to their values at the time of roll through zero, t_z. In order to estimate the integrals in Eqs. (6) and (7), we let

$$\pi s^2 = (qAd/I_x)C_l(t - t_z)^2 \tag{8}$$

Then,

$$\dot{z}_\beta = \chi[(\beta_t\cos P_z - \alpha_t\sin P_z)I_1 - (\beta_t\sin P_z + \alpha_t\cos P_z)I_2] \tag{9}$$

$$\dot{z}_\alpha = \chi[(\alpha_t\cos P_z + \beta_t\sin P_z)I_1 - (\alpha_t\sin P_z - \beta_t\cos P_z)I_2] \tag{10}$$

where

$$P_z = \int_0^{t_z} p(t)dt$$

$$\chi = (C_N\dot{z} - C_A)/m[\pi I_xAq/dC_l]^{1/2} \tag{11}$$

$$I_1 = \int_{-\infty}^{\infty}\cos\frac{\pi}{2}\,s^2ds \qquad \text{and} \qquad I_2 = \int_{-\infty}^{\infty}\sin\frac{\pi}{2}\,s^2ds \tag{12}$$

The limits of integration for I_1 and I_2 are set $+\infty$ and $-\infty$ because the contribution from the large values of s is small. The integrals I_1 and I_2 are the Fresnel integrals, and each has the value of unity.

The velocity perturbation $|\dot{z}|$ normal to the body velocity is then given by

$$|\dot{z}| = (\dot{z}_\alpha^2 + \dot{z}_\beta^2)^{1/2} = \chi[2(\beta_t^2 + \alpha_t^2)]^{1/2} \tag{13}$$

The perturbation in flight path angle γ_z due to roll reversal is then

$$\gamma_z = |\dot{z}|/V \tag{14}$$

Hence the semiminor axis, b_z, and the semimajor axis, a_z, of

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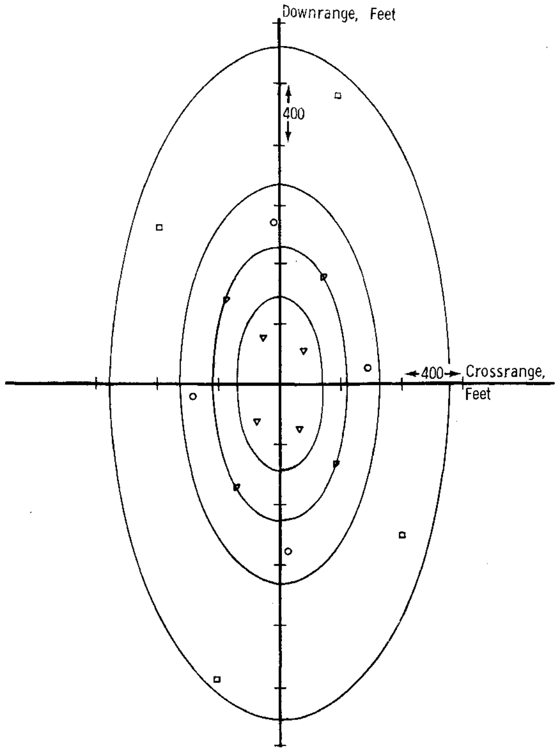

Fig. 1 **Comparison of 6 DOF and analytical results (same legend as in Fig. 2).**

the dispersion ellipse can be approximated by

$$b_z = \frac{h_z \gamma_z}{\sin\gamma} = \frac{(C_{N\alpha} - C_A)h_z}{m \sin\gamma}\left[\frac{\pi I_x \rho_z A(\beta_t^2 + \alpha_t^2)}{dC_l}\right]^{1/2} \quad (15)$$

$$a_z = b_z/\sin\gamma \quad (16)$$

where γ is the flight path angle, h_z is the altitude of roll reversal, and ρ_z is the atmospheric density at h_z. If C_A and β_t are set equal to zero, then Eq. (15) is the same as that of Ref. 2. The estimate for a_z of Ref. 1 is 15.4% lower than the above estimate because Fuess integrated I_1 and I_2 over two cycles only. The results of Fuess can overestimate or underestimate the magnitude of the dispersion.[1]

For a varying dynamic pressure but constant C_l, Eq. (2) can be integrated if we assume an exponentially varying atmosphere and use the straightline trajectory of Allen and Eggers.[4] The result of integration is

$$p = -(Ad\beta V_i/I_x)C_l(e^{-\nu\rho} - e^{-\nu\rho_i}) + p_i \quad (17)$$

Here, V_i, ρ_i, and p_i are the initial velocity, atmospheric density, and roll rate, respectively, and

$$\beta = m/C_A A,\ \nu = k/2\beta \sin\gamma \quad (18)$$

where k is the atmospheric scale height. Then,

$$h_z = -k \ln\left[-\frac{2}{\nu\rho_0}\ln\left(\frac{p_i I_x}{Ad\beta V_i C_l} + e^{-\nu\rho_i}\right)\right] \quad (19)$$

where ρ_0 is the sea level atmospheric density.

Comparison with Six-Degree-of-Freedom Calculations

Figures 1 and 2 compare the analytical results with six-degree-of-freedom (6 DOF) numerical calculations. All of the calculations were made for m = 5.122 slugs, A = 1.485 ft^2, d = 1.375 ft, I_x = 0.43 slug-ft^2, $C_{N\alpha}$ = 1.93/rad., h_i = 100 kft, V_i = 21,000 fps, γ = 30°, and p_i = 12.56 rad/sec. In the calculations, the ballistic coefficient, the c.g. offset, the trim components β_t and α_t, and the roll orientation were varied.

For a given body and trajectory parameter, Fig. 1 shows the dispersion ellipse as calculated from Eqs. (15) and (16).

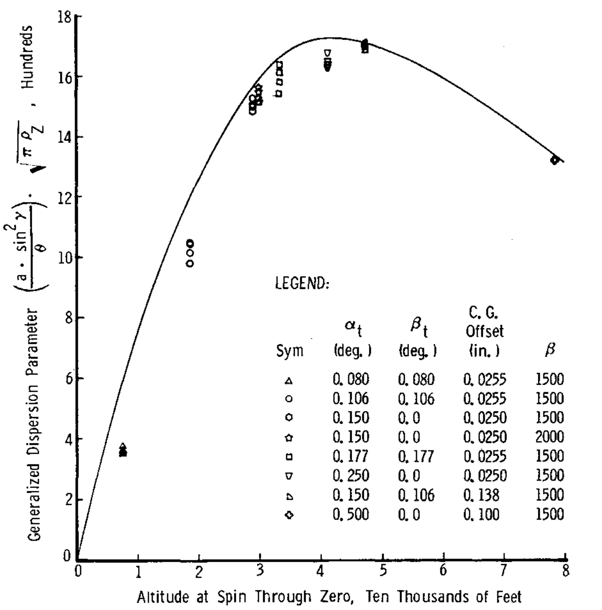

Fig. 2 **Maximum dispersion variation with altitude at spin through zero.**

For each case, Fig. 1 also shows four data points (corresponding to four different roll orientations) obtained from 6 DOF calculations. It shows also that the analytical results provide an upper bound in each case.

Figure 2 shows the variation of the maximum dispersion generalized parameter with the altitude of roll reversal. Six DOF results are shown also for comparison. Twenty-nine different 6 DOF points are shown for eight different trajectory and body parameters. Except in one case, four different roll orientations were used.‡ The cross range (CR) and delta range (ΔR) dispersions are then used to calculate a_z assuming that (ΔR,CR) is a point on the ellipse. Thus,

$$a_z = [(\Delta R)^2 + (CR)^2/\sin^2\gamma]^{1/2} \quad (20)$$

This figure also shows good agreement between the analysis and the 6 DOF calculations with the analysis always providing an upper bound. Figure 2 shows also that the accuracy of the analytical results improves as the altitude of roll reversal increases.

Concluding Remarks

An analytical estimation of the dispersion due to roll reversal is presented. The analysis is limited to cases where the following conditions hold: 1) a single roll reversal occurs; 2) the roll rate is away from resonance; 3) a high ballistic coefficient; and 4) small asymmetrics. Equation (15) shows that, as a function of h_z, the impact point dispersion is maximum where $h_z = 2k$.

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‡ The 6 DOF results showed that the combination of an α_t = 0.5° and a c.g. offset of 0.10 in. at all altitudes produced a roll reversal at h_z = 78,400 ft and a condition of steady-state roll resonance at the lower altitudes. To determine the dispersion due to roll reversal, in this case, α_t is set to equal zero below 70,000 ft.